[Historical Perspective (census.gov)](https://www.census.gov/topics/public-sector/congressional-apportionment/about/historical-perspective.html)

The primary reason for the establishment of the **decennial census of population** is set forth in **Article 1, Section 2, of the Constitution**.

The Constitution provides for an **enumeration of the population** to serve as the **basis** for the apportionment of members of the U.S. House of Representatives among the states, with the provision that **each state must have at least one representative**.

From 1790 to the present, an apportionment has been made on the basis of each census, except following the census of 1920.

Calculation of a Congressional apportionment requires **three factors**—

* the apportionment population of each state,
* the number of representatives to be allocated among the states,
* and a method to use for the calculation.

From **1800 through 1840**, the number of representatives was determined by the ratio of the number of persons each was to represent ("**fixed ratio**"), although the way to handle fractional remainders changed. Therefore, **the number of representatives changed** with that ratio, as well as with population growth and the admission of new states.

For the **1850 census and later** apportionments, the number of seats was determined prior to the final apportionment ("**fixed house size**"); and thus, the ratio of persons each was to represent was the result of the calculations. In **1911**, the House size was fixed at **433** with provision for the addition of one seat each for Arizona and New Mexico when they became states (U.S. Statutes at Large, 37 Stat 13, 14 (1911)). The **House size, 435** members, has been **unchanged since**, except for a **temporary increase to 437** **at the time of admission of Alaska and Hawaii as states (following the 1950 census).-**

**Method of Apportionment**

It is impossible to attain absolute mathematical equality in terms of the **number of persons per representative**, or in the share each person has in a representative, when seats are to be apportioned among **states of varying population size** and when there **must be a whole number of representatives per state**. Proportional voting (fractional seats) has never been attempted in the U.S. House of Representatives. Laws concerning the method of apportionment are **codified in the United States Code, Title 2**.

The Constitution set the number of representatives at **65** **from 1787 until the first enumeration in 1790.**

**1790 to 1830 (fixed ratio with rounding down)**  
The "**Jefferson method**" of greatest divisors (**fixed ratio with rejected fractional remainders**). Under this method, a ratio of persons to representatives was selected; the population of each state was divided by that number of persons. The resulting whole number of the quotient was the number of representatives each state received. Fractional remainders were not considered, no matter how large. Thus a state with a quotient of 3.99 received three representatives, the same number as a state with a quotient of 3.01. The size of the House of Representatives was not predetermined, but resulted from the calculation.

The rule typically gives large parties an excessive number of seats, with their seat share generally exceeding the ideal share rounded up.[[6]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:0222-6): 81

The first apportionment, based on the **1790** census, resulted in 105 members.

**1840 (fixed ratio with rounding (regular)**  
The "**Webster method**" of major fractions (fixed ratio with retained major fractional remainders). This method was applied in the same way as the Jefferson method, except if a fractional remainder were greater than one-half, another seat would be assigned. Thus a state with a quotient of 3.51 received four representatives, while a state with a quotient of 3.49 received three. In this method also, the size of the House of Representatives was not predetermined but resulted from the calculation. [Daniel Webster's](https://en.wikipedia.org/wiki/Daniel_Webster) method uses the fencepost sequence post(*k*) = *k*+.5 (i.e. 0.5, 1.5, 2.5); this corresponds to the standard [rounding rule](https://en.wikipedia.org/wiki/Rounding). Equivalently, the odd integers (1, 3, 5…) can be used to calculate the averages instead.[[12]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:42-12)[[20]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-20)

Adams' apportionment never exceeds the upper end of the [ideal frame](https://en.wikipedia.org/wiki/Quota_rule), and minimizes the worst-case underrepresentation.[[12]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:42-12) However, violations of the [lower seat quota](https://en.wikipedia.org/wiki/Quota_rule) are common.[[18]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-18) Like Jefferson, Adams' method performs poorly according to most metrics of proportionality.[[16]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:03-16)

**1850 to 1900 (largest remainder)**  
The "**Vinton" or "Hamilton" method** established a predetermined number of representatives for each apportionment, and divided the population of each state by a ratio determined by dividing the apportionment population of the United States by the total number of representatives. The resulting whole number was assigned to each state, with an additional seat assigned, one at a time, to the states with the largest fractional remainders, up to the predetermined size of the House of Representatives. This method was subject to the "**Alabama paradox**," in which a state could receive fewer representatives if the size of the House of Representatives was increased.

**1910, 1930**  
The **method of major fractions** assigned seats **similarly to the Webster method** of 1840 by **rounding fractional remainders using the arithmetic mean**. The ratio was selected so that the result would be the predetermined size of the House of Representatives. In 1910, the House size was fixed at 433 with provision for the addition of one seat each for Arizona and New Mexico when they became states.

**1940 to Present (method of equal proportions)**

The current method used, the Method of Equal Proportions, was adopted by congress in 1941 following the census of 1940.  
The method of equal proportions assigns seats **similarly to the Jefferson and Webster method**, except it **rounds fractional remainders** of the quotient of the state population divided by the ratio **differently**. **With this method, an additional seat is assigned if the fraction exceeds the difference obtained by subtracting the integer part of the quotient from the geometric mean of this integer and the next consecutive integer.** The **size of the House of Representatives remained fixed at 435** (**except when Alaska and Hawaii became states, there was a temporary addition of one seat for each until the apportionment following the 1960 census**).

Following the 1990 census, two lawsuits concerning apportionment issues were filed in federal courts. The U.S. Supreme Court held that the method of equal proportions was constitutional; that the Congress had properly exercised its apportionment authority; and that the inclusion of U.S. federal military and civilian personnel, and their dependents, in the apportionment populations of the states was constitutional. These cases were United States Department of Commerce v. Montana 112 S.Ct. 1415 (1992) and Franklin v. Massachusetts 112 S.Ct. 2767 (1992).

Hill's method tends to produce very similar results to Webster's method; when first used for [congressional apportionment](https://en.wikipedia.org/wiki/Congressional_apportionment), the two methods differed only in whether they assigned a single seat to [Michigan](https://en.wikipedia.org/wiki/Michigan) or [Arkansas](https://en.wikipedia.org/wiki/Arkansas).[[6]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:0222-6): 58

For more specific information on the equal proportions method, see [Computing Apportionment](https://www.census.gov/topics/public-sector/congressional-apportionment/about/computing.html).

[Computing Apportionment (census.gov)](https://www.census.gov/topics/public-sector/congressional-apportionment/about/computing.html)

This method assigns seats in the House of Representatives according to a "**priority" value**. The priority value is determined by multiplying the population of a state by a "multiplier."

Each of the 50 states is given one seat out of the current total of 435. The next, or 51st seat, goes to the state with the highest priority value and becomes that state's second seat. This continues until all 435 seats have been assigned to a state. This is how it is done.

**Equal Proportions Method (Huntington-Hill method): highest averages method**

With the highest averages algorithm, every party begins with 0 seats. Then, at each iteration, we allocate a seat to the party with the *highest vote average,* i.e. the party with the most [votes per seat](https://en.wikipedia.org/wiki/Seats-to-votes_ratio)*.* This method proceeds until all seats are allocated.[[12]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:42-12)

**The method minimizes the**[**relative difference**](https://en.wikipedia.org/wiki/Relative_difference)**in the number of constituents represented by each legislator. In other words, the method selects the algorithm such that no transfer of a seat from one state to another can reduce the percent error in representation for both states.**[**[1]**](https://en.wikipedia.org/wiki/Huntington%E2%80%93Hill_method#cite_note-:0-1)In this method, as a first step, each of the 50 states is given its one guaranteed seat in the House of Representatives, leaving 385 seats to assign. The remaining seats are allocated one at a time, to the state with the highest average district population, to bring its district population down. However, it is not clear if we should calculate the average *before* or *after* allocating an additional seat, and the two procedures give different results. Huntington-Hill uses a [continuity correction](https://en.wikipedia.org/wiki/Continuity_correction) as a compromise, given by taking the [geometric mean](https://en.wikipedia.org/wiki/Geometric_mean) of both divisors, i.e.:[[4]](https://en.wikipedia.org/wiki/Huntington%E2%80%93Hill_method#cite_note-:422-4)

An=Pn(n+1) A square and triangle equation

Description automatically generated with medium confidence

where *P* is the population of the state, and *n* is the number of seats it currently holds before the possible allocation of the next seat.

**Hill's (Huntington–Hill) method**

[[edit](https://en.wikipedia.org/w/index.php?title=Highest_averages_method&action=edit&section=9)]

*Main article:*[*Huntington–Hill method*](https://en.wikipedia.org/wiki/Huntington%E2%80%93Hill_method)

In the [Huntington–Hill method](https://en.wikipedia.org/wiki/Huntington%E2%80%93Hill_method), the signpost sequence is post(*k*) = √*k* (*k*+1), the [geometric mean](https://en.wikipedia.org/wiki/Geometric_mean) of the neighboring numbers. **Conceptually, this method rounds to the integer that has the smallest**[**relative (percent) difference**](https://en.wikipedia.org/wiki/Relative_change#Logarithmic_change)**.** For example, the difference between 2.47 and 3 is about 19%, while the difference from 2 is about 21%, so 2.47 is rounded up. This method is used for allotting seats in the US House of Representatives among the states.[[12]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:42-12)

Hill's method tends to produce very similar results to Webster's method; when first used for [congressional apportionment](https://en.wikipedia.org/wiki/Congressional_apportionment), the two methods differed only in whether they assigned a single seat to [Michigan](https://en.wikipedia.org/wiki/Michigan) or [Arkansas](https://en.wikipedia.org/wiki/Arkansas).[[6]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:0222-6): 58

Consider the reapportionment following the 2010 U.S. census: after every state is given one seat:

1. The largest value of *A*1 corresponds to the largest state, California, which is allocated seat 51.
2. The 52nd seat goes to Texas, the 2nd largest state, because its *A*1 priority value is larger than the *An* of any other state.
3. The 53rd seat goes back to California because its *A*2 priority value is larger than the *An* of any other state.
4. The 54th seat goes to New York because its *A*1 priority value is larger than the *An* of any other state at this point.

This process continues until all remaining seats are assigned. Each time a state is assigned a seat, *n* is incremented by 1, causing its priority value to be reduced.

**Examples**

[[edit](https://en.wikipedia.org/w/index.php?title=Huntington%E2%80%93Hill_method&action=edit&section=3)]

Each eligible party is assigned one seat. With all the initial seats assigned, the remaining five seats are distributed by a priority number calculated as follows. Each eligible party's (Parties A, B, and C) total votes is divided by √2  • 1 ≈ 1.41, then by approximately 2.45, 3.46, 4.47, 5.48, 6.48, 7.48, and 8.49. The 5 highest entries, marked with asterisks, range from **70,711** down to **28,868**. For each, the corresponding party gets another seat.

Original P\_a=100 000 P\_b= 80 000 P\_c = 30 000 P\_a+P\_b+P\_c=210 000 seats=8

Ideal seats =P\_i/P\*8

A screenshot of a calculator

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P - represents a state's total population

n - represents the number of seats a state would have if it gained a seat (because all states automatically received one seat the next seat gained is "seat two," and the next "seat three," and the next "seat four," and so on.)

**The multiplier equals:**  
1 divided by the square root of n(n-1)  
[which is called the reciprocal of the geometric mean]. Computing these values is quite easy using a computer with spreadsheet software (such as Excel).

**Thus the formula for calculating the multiplier for the second seat is:**  
1 divided by the square root of 2(2-1)  
or 1/1.414213562 or 0.70710678

**the multiplier for the third seat is:**  
1 divided by the square root of 3(3-1)  
1/2.449489743 or 0.40824829

**the multiplier for the fourth seat is:**  
1 divided by the square root of 4(4-1)  
1/3.464101615 or 0.288675134

Continue until an appropriate number of multipliers have been calculated.

Once the "multipliers" have been calculated, the next step is to multiply this figure by the population total for each of the 50 states (the District of Columbia is not included in these calculations). The resulting numbers are the priority values. Make sure you compute enough multipliers and priority values to cover the largest number of seats in the U.S. House of Representatives that any one state stands to gain. For example, if the largest number of seats currently assigned to a state is 60, multipliers and priority values must be calculated for at least the 60th seat. If you are using a computer, you should compute multipliers for seats 2 through 70. This will assure you have enough multipliers and priority values for apportionment.

Once you've calculated priority values for the total number of potential seats for each state, the next step is to rank and assign seat numbers to the resulting priority values starting with seat 51, until all 435 seats have been assigned (remember, each state automatically received one seat). Next, tally the number of seats for each state to arrive at the total number of seats in the House of Representatives apportioned to each state.

[Huntington–Hill method - Wikipedia](https://en.wikipedia.org/wiki/Huntington%E2%80%93Hill_method)

[Highest averages method - Wikipedia](https://en.wikipedia.org/wiki/Highest_averages_method)

The **highest averages**, **divisor**, or **divide-and-round methods**[[1]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:422-1) are a family of [apportionment](https://en.wikipedia.org/wiki/Apportionment_(politics)) algorithms that aim to [fairly divide](https://en.wikipedia.org/wiki/Fair_division) a legislature between several groups, such as [political parties](https://en.wikipedia.org/wiki/Political_party) or [states](https://en.wikipedia.org/wiki/State_(sub-national)).[[1]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:422-1)[[2]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:432-2) More generally, divisor methods can be used to round shares of a total, e.g. percentage points (which must add up to 100).[[2]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:432-2)

The methods aim to treat voters equally by ensuring legislators [represent an equal number of voters](https://en.wikipedia.org/wiki/One_man,_one_vote) by ensuring every party has the same [seats-to-votes ratio](https://en.wikipedia.org/wiki/Seats-to-votes_ratio) (or *divisor*).[[3]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:02222-3): 30 Such methods divide the number of votes, by the number of votes-per-seat, then round the total to get the final apportionment. In doing so, the method approximately maintains [proportional representation](https://en.wikipedia.org/wiki/Proportional_representation), so that a party with e.g. twice as many votes as another should win twice as many seats.[[3]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:02222-3): 30

The divisor methods are generally preferred by [social choice theorists](https://en.wikipedia.org/wiki/Social_choice_theory) to the [largest remainder methods](https://en.wikipedia.org/wiki/Largest_remainder_method), as they produce more-proportional results by most metrics and are less susceptible to [apportionment paradoxes](https://en.wikipedia.org/wiki/Apportionment_paradox).[[4]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:122-4)[[5]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:522-5)[[6]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:0222-6) In particular, divisor methods satisfy [vote-ratio monotonicity](https://en.wikipedia.org/wiki/Vote-ratio_monotonicity) and [participation](https://en.wikipedia.org/wiki/Participation_criterion), i.e. voting *for* a party can never cause it to *lose* seats, unlike in the [largest remainders methods](https://en.wikipedia.org/wiki/Largest_remainders_method); in addition, they are not sensitive to [spoiler effects](https://en.wikipedia.org/wiki/Spoiler_effect).[[5]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:522-5)

**Divisor procedure**

The two names for these methods—highest averages and divisors—reflect two different ways of thinking about them, and their two independent inventions. However, both procedures are equivalent and give the same answer.[[10]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:423-10)

Divisor methods are based on [rounding](https://en.wikipedia.org/wiki/Rounding) rules, defined using a [*signpost sequence*](https://en.wikipedia.org/wiki/Signpost_sequence) post(*k*)*,* where *k* ≤ post(*k*) ≤ *k*+1*.* Each signpost marks the boundary between natural numbers, with numbers being rounded down if and only if they are less than the signpost.[[11]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:433-11)

The divisor procedure apportions seats by searching for a *divisor* or [*electoral quota*](https://en.wikipedia.org/wiki/Electoral_quota). This divisor can be thought of as the number of votes a party needs to earn one additional seat in the legislature, the ideal population of a [congressional district](https://en.wikipedia.org/wiki/Constituency), or the number of voters represented by each legislator.[[12]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:42-12)

If each legislator represented an equal number of voters, the number of seats for each state could be found by dividing the population by the divisor.[[12]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:42-12) However, seat allocations must be whole numbers, so to find the apportionment for a given state we must round (using the signpost sequence) after dividing. Thus, each party's apportionment is given by:[[12]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:42-12)

seats=round⁡(votesdivisor)

Usually, the divisor is initially set to equal the [Hare quota](https://en.wikipedia.org/wiki/Hare_quota). However, this procedure may assign too many or too few seats. In this case the apportionments for each state will not add up to the total legislature size. A feasible divisor can be found by [trial and error](https://en.wikipedia.org/wiki/Root-finding_algorithms).[[13]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-13)

The Hare quota may be given as:

totalvotestotalseats

where

* **Total votes** = the total valid poll; that is, the number of valid (unspoilt) votes cast in an election.
* **Total seats** = the total number of seats to be filled in the election.

With the highest averages algorithm, every party begins with 0 seats. Then, at each iteration, we allocate a seat to the party with the *highest vote average,* i.e. the party with the most [votes per seat](https://en.wikipedia.org/wiki/Seats-to-votes_ratio)*.* This method proceeds until all seats are allocated.[[12]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:42-12)

However, it is unclear whether it is better to look at the vote average *before* assigning the seat, what the average will be *after* assigning the seat, or if we should compromise with a [continuity correction](https://en.wikipedia.org/wiki/Continuity_correction). These approaches each give slightly different apportionments.[[12]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:42-12) In general, we can define the averages using the signpost sequence:

average:=votespost⁡(seats)

While all divisor methods share the same general procedure, they differ in the choice of signpost sequence and therefore rounding rule. Note that for methods where the first signpost is zero, every party with at least one vote will receive a seat before any party receives a second seat; in practice, this typically means that every party must receive at least one seat, unless disqualified by some [electoral threshold](https://en.wikipedia.org/wiki/Electoral_threshold).[[14]](https://en.wikipedia.org/wiki/Highest_averages_method#cite_note-:43-14)

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